

Social Democratic Horizon (4): Rebuilding Social Democracy

APPENDICES A2-A4

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APPENDIX A2 – Associative Democratic Model

A Provisional Technical Framework

The purpose of this appendix is not to provide a completed electoral system. It is to demonstrate that participation through overlapping Associations can be represented and measured in several possible ways.

The formulas are provisional. Consultation, representation on an administrative body, municipal participation, and allocation of seats in a complementary Associative chamber are not necessarily the same problem. Different purposes may require different measurements.

Ordinary parliamentary elections would initially remain governed by equal citizenship and “one person, one vote.” The quantities introduced below concern Associative participation or influence, not additional votes in ordinary elections.

Throughout this appendix, subscripts identify individuals or Associations. Superscripts written as words are descriptive labels, not powers. The summation sign, $\sum$, means that the specified quantities are added together.

A2.1 – Individuals and Associations

Assume that the population under examination contains N individuals and M Associations.

Individuals are indexed by $i = 1, \dots, N$, and Associations by $a = 1, \dots, M$. Define the membership indicator x_{ia} as:

$$x_{ia} = \begin{cases} 1, & \text{if person } i \text{ is a registered member of Association } a, \\ 0, & \text{otherwise.} \end{cases}$$

The number of Associations to which person i belongs is:

$$k_i = \sum_{a=1}^M x_{ia}.$$

Here, k_i is a membership count. A person with no registered memberships has $k_i = 0$.

Membership alone does not demonstrate activity, agreement with every position taken by an Association, or independence from other Associations. These indicators record only one dimension: verified membership.

A2.2 – The Equal-Budget Model

The simplest model gives every individual one unit of available Associative weight. Each person may distribute that unit among the Associations they choose to support.

Let f_{ia} be the fraction of person i 's unit assigned to Association a , subject to:

$$f_{ia} \geq 0, \quad \sum_{a=1}^M f_{ia} \leq 1.$$

In this membership-based version, weight may be assigned only to Associations in which the person is registered:

$$f_{ia} = 0 \text{ when } x_{ia} = 0.$$

The inequality permits a person to leave some or all of their available weight unassigned.

If a person divides the entire unit equally among their Associations, then:

$$f_{ia} = \frac{x_{ia}}{k_i}, \quad k_i > 0.$$

For a person with no memberships, all f_{ia} are zero.

A person belonging to four Associations would therefore allocate one quarter of their unit to each, unless another distribution were chosen. More memberships would not increase the person's total formal Associative weight.

Association a 's total assigned weight, denoted by W_a , is:

$$W_a = \sum_{i=1}^N f_{ia}.$$

This model prevents multiple memberships from producing additional formal weight. A person can express several parts of their identity without acquiring a larger total budget than someone with one membership.

Its limitation is that nominal and active members may count equally. Whether activity should affect influence is a separate normative question.

A2.3 – A Participation-Intensity Variant

An alternative could allow participation in several Associations to produce some additional Associative influence, while applying diminishing returns.

For exploratory purposes, person i 's participation score, v_i , might be:

$$v_i = \begin{cases} 1 + \alpha(\sqrt{k_i} - 1), & k_i \geq 1, \\ 0, & k_i = 0, \end{cases} \quad 0 \leq \alpha \leq 1.$$

The parameter α controls how strongly additional memberships increase the score.

When $\alpha = 0$, every person with at least one membership receives a score of one. When $\alpha = 1$, the score equals the square root of the membership count.

A person with four memberships would therefore receive a score of two when $\alpha = 1$, rather than four. Intermediate values of α produce smaller increases.

There is nothing privileged about the square-root function. Logarithmic or other concave functions could also represent diminishing returns.

This is an illustration of how breadth of membership might be measured. It does not itself measure actual participation intensity, nor does it establish that this score should determine institutional influence.

The risks are obvious. Wealthy or highly educated people may find it easier to join many Associations; nominal memberships may be collected to increase influence; and organisations may coordinate membership campaigns. The score may reward quantity of membership rather than quality of participation.

For these reasons, the equal-budget model provides the safer starting point.

A2.4 – Activity and Retention

If activity is included, assigned weight could be multiplied by an activity factor r_{ia} :

$$0 \leq r_{ia} \leq 1.$$

Here, r_{ia} represents a specified measure of person i 's activity in Association a during a stated assessment period. If the measure concerns retention instead, its definition and observation period must be stated separately.

Association a 's activity-adjusted weight is:

$$W_a^{\text{act}} = \sum_{i=1}^N f_{ia} r_{ia}.$$

The superscript “act” distinguishes this quantity from the unadjusted weight W_a .

Possible indicators include participation in meetings, elections, consultations, working groups, or other defined activities. Each creates difficulties. Attendance may favour people with more available time; online activity may favour those with greater digital access; and financial contributions would reproduce economic inequality. Records may also be manipulated.

No activity measure should therefore be adopted merely because it can be calculated. It must be transparent, auditable, and appropriate to the institutional purpose.

A2.5 – Overlapping Associations

The equal-budget model prevents one person's full unit from being counted repeatedly across Associations. It does not eliminate every form of overlap or dependence.

Two Associations may have nearly identical memberships. Several organisations may share leadership, financing, or coordinated decision-making. Their formal number may therefore exaggerate their effective pluralism.

Membership similarity can help identify overlap, but overlap does not necessarily make an Association redundant. Organisations with similar memberships may address different issues. Conversely, organisations with different memberships may depend upon the same centre of power.

Membership overlap, common leadership, common financing, and coordinated decisions should therefore be examined separately where possible.

A2.6 – Concentration of Associative Weight

Association a 's share of total assigned weight, p_a , is:

$$p_a = \frac{W_a}{\sum_{b=1}^M W_b}.$$

The index b runs over the same Associations as a ; it is used separately to identify the denominator's summation. This calculation requires positive total weight.

A simple effective number of weighted Associations is:

$$N_{\text{Association}}^{\text{eff}} = \frac{1}{\sum_{a=1}^M p_a^2}.$$

The superscript “eff” means effective. This is the number of equally weighted Associations that would produce the same concentration.

If ten Associations share the weight equally, the effective number is ten. If one holds most of it, the effective number falls. Where activity-adjusted weights are used instead, replace W_a by W_a^{act} and label the result accordingly.

This quantity measures the concentration of calculated Associative weight. It does not measure the number of individual decision-makers, internal democracy, or effective independence.

The broader Effective Number of Decision-Makers (**ENDM**) must also consider inequality of influence, concentration of leadership, common financing, internal democracy, and whether participation can affect decisions.

A2.7 – Institutional Use

Association weights should not automatically be translated into seats in the existing parliament.

Possible uses include allocating consultation time, selecting advisory representatives, distributing limited seats on administrative bodies, allocating participatory support, or comparing organised participation over time.

If seats are eventually allocated in a complementary body, conventional proportional methods such as Sainte-Laguë¹ could be applied to the weights. This is a later institutional choice, not a necessary component of **ADM**.

Any allocation must preserve constitutional protections, minority rights, and clear accountability. People who do not join Associations must retain accessible channels for presenting views and challenging decisions.

A2.8 – What the Formulas Can and Cannot Show

¹ The Sainte-Laguë method is a highest-averages method for allocating seats proportionally. Each party's or Association's total is divided successively by the odd numbers 1, 3, 5, 7, and so forth, and seats are assigned according to the resulting quotients. Its use here would concern only the possible allocation of seats in a future complementary body, not ordinary parliamentary elections or the calculation of Association weights (Sainte-Laguë, 1910).

The formulas can describe membership, the distribution of individual support, activity-adjusted participation, and the concentration of Associative weight.

They cannot establish whether an Association is correct, democratic, well informed, independent, or entitled to a particular share of authority. Nor can they demonstrate that participation improves a decision.

They should be treated as components of an instrument panel. No single gauge describes the whole system. Several partial measures may help us observe whether organised participation is becoming broader, more equal, and more independent—or narrower, more unequal, and more concentrated.

The political goal comes first. Measurement helps evaluate movement towards or away from that goal; it does not define the goal on its own.

APPENDIX A3 – Democratic Participation

Effective Number of Active Participants

The Effective Number of Active Participants (**ENAP**) is intended to describe the scale and continuity of civic participation between elections.

It does not measure the quality of decisions, the independence of Associations, or equality of influence. It asks a narrower question: how many distinct people participate actively during successive periods, and how continuous is that participation?

Throughout this appendix, subscripts identify Associations or time periods. Superscripts such as “sample” and “rough” are descriptive labels, not powers. The subscript H denotes a harmonic mean, and an asterisk identifies an overlap-corrected version. The summation sign, $\sum$, means that the specified quantities are added together.

A3.1 – Defining Active Participation

Let time be divided into T periods, indexed by:

$$t = 1, \dots, T.$$

Here, T is the total number of periods examined. A period might be a month, quarter, or year, depending upon the institution being studied.

Let M be the number of Associations included, indexed by:

$$a = 1, \dots, M.$$

For each period, define:

- $m_{a,t}$: the number of registered members of Association a ;
- $r_{a,t}$: the proportion of those members classified as active, where $0 \leq r_{a,t} \leq 1$;

- A_t : the total number of active membership records across all included Associations;
- U_t : the number, or estimated number, of unique active persons after correcting for individuals active in more than one Association.

The total number of active membership records is:

$$A_t = \sum_{a=1}^M m_{a,t} r_{a,t}.$$

Each person is counted at most once within an Association during a period, but may appear in several Associations' records. An Association with no members contributes zero.

Before this quantity can be interpreted, "active" must be defined. Possible criteria include participation in a meeting, election, consultation, working group, or another specified activity. Different definitions will produce different results.

Merely paying a membership fee may be sufficient for some purposes but not others. Attendance at one meeting may demonstrate more involvement than nominal membership, but it does not necessarily measure the quality or influence of participation.

The definition of activity must therefore accompany every reported **ENAP**.

A3.2 – Correcting for Overlapping Membership

A person may participate actively in several Associations during the same period. Simply adding active membership records would count that person several times.

For periods with $A_t > 0$, define a deduplication factor δ_t , where:

$$0 < \delta_t \leq 1.$$

This factor represents the ratio of unique active persons to active membership records. When estimated rather than directly measured, it gives:

$$U_t \approx \delta_t A_t,$$

or equivalently:

$$U_t \approx \delta_t \sum_{a=1}^M m_{a,t} r_{a,t}.$$

The symbol $\approx$ means approximately equal.

If a suitable sample provides both counts, the factor may be estimated as:

$$\delta_t \approx \frac{U_t^{\text{sample}}}{A_t^{\text{sample}}},$$

where:

- U_t^{sample} is the number of distinct active people in the sample;
- A_t^{sample} is the number of active membership records associated with those sampled observations.

The sampling design must be suitable for estimating overlap in the population being studied.

If everyone is active in only one Association, $\delta_t = 1$. If people are active in several Associations, it becomes smaller. When $A_t = 0$, set $U_t = 0$; no deduplication factor is needed.

Directly linking records creates privacy risks. Where feasible, deduplication should use privacy-preserving record linkage, independent statistical institutions, or suitable samples.² Associations should not be required to disclose identifiable membership lists to one another.

Capture–recapture methods might be explored under particular sampling conditions, but they are not universally suitable. Their estimates can be affected seriously by unequal inclusion probabilities and dependence between membership lists.³

A3.3 – Participation During One Period

The simplest measure of effective participation during period t is:

$$N_t^{\text{part}} = U_t.$$

Here, N_t^{part} denotes the period-specific participation count. The superscript “part” is a label meaning participation.

This quantity counts unique active persons. It does not imply that all participants had equal influence or acted independently.

A person who attended a meeting without an opportunity to speak and someone who controlled its agenda may both count as participants. Their unequal influence requires separate measurement.

A3.4 – Participation Across Time

Participation may fluctuate considerably. An organisation may mobilise many people during an election or crisis and become nearly inactive afterwards.

If the interest is average participation, use the arithmetic mean:

² Vatsalan et al. (2013).

³ Hook & Regal (1995).

$$\bar{U} = \frac{1}{T} \sum_{t=1}^T U_t.$$

The symbol $\bar{U}$ denotes the arithmetic mean of period-specific unique-participant counts. Every period receives equal weight.

If the interest is continuity and sensitivity to periods of low participation, a harmonic mean may be useful:

$$\text{ENAP}_H = \frac{T}{\sum_{t=1}^T \frac{1}{U_t}}, \quad U_t > 0 \text{ for every period.}$$

The subscript H identifies the harmonic-mean version of **ENAP**. Low-participation periods reduce this measure more strongly than they reduce the arithmetic mean.

For this appendix, if any included period has zero participation, the continuity-sensitive index is assigned zero. This is an explicit convention extending the positive-count formula.

The use of a harmonic mean resembles its use in some models of effective population size, where small-population periods can have disproportionate long-term effects. The political consequences are different. The analogy explains the choice of a measure sensitive to low periods; it does not establish a biological equivalence.

Both means may be reported:

- the arithmetic mean describes average participation;
- the harmonic mean describes participation with a stronger penalty for low periods;
- their difference indicates unevenness over time.

Neither mean identifies the order of events. The same values can describe rising, falling, or irregular participation. A time series should therefore accompany them where that distinction matters.

Every **ENAP** should state the period length, the mean used, and the reason for choosing it.

A3.5 – Minimal-Data Version

If overlap information is unavailable, a rough estimate can be calculated from total membership records and activity rates.

Define:

- M_t : the total number of registered membership records during period t ;
- r_t : the estimated proportion of those records classified as active.

The symbol M_t counts membership records, whereas M in A3.1 counts Associations. Thus:

$$M_t = \sum_{a=1}^M m_{a,t}.$$

For $M_t > 0$, the overall activity proportion should be weighted by membership counts:

$$r_t = \frac{\sum_{a=1}^M m_{a,t} r_{a,t}}{M_t}.$$

It should not be an unweighted average of Association activity rates.

An uncorrected approximation is:

$$U_t^{\text{rough}} \approx M_t r_t.$$

The superscript “rough” identifies an approximation without correction for overlapping active memberships. Strictly speaking, the product estimates active membership records rather than distinct people.

A stability-sensitive version is:

$$\text{ENAP}_H^{\text{rough}} = \frac{T}{\sum_{t=1}^T \frac{1}{M_t r_t}}.$$

This may substantially overestimate unique participation and must be labelled as uncorrected.

With estimated overlap factors, define:

$$\text{ENAP}_H^* = \frac{T}{\sum_{t=1}^T \frac{1}{\delta_t \sum_{a=1}^M m_{a,t} r_{a,t}}}.$$

The asterisk denotes overlap correction. These formulas require positive period counts; a zero-count period receives the zero-index treatment stated in **A3.4**.

Uncertainty in activity rates and overlap factors should not disappear behind an apparently precise number. Where possible, report a plausible range.

A3.6 – Participation Quality and Influence

The participation count could be multiplied by a specified quality or intensity factor:

$$Q_t = \kappa_t U_t, \quad 0 \leq \kappa_t \leq 1.$$

Here:

- Q_t is an adjusted participation score, not a headcount;
- κ_t is a factor representing an explicitly defined feature of participation during period t .

The factor might concern access to relevant information or satisfaction of a specified procedural requirement. Its construction must be explained.

However, “quality” is not one-dimensional. Deliberative depth, representativeness, information, equality of speaking time, independence, and actual influence are different characteristics. Combining them in one coefficient may hide more than it reveals.

For most purposes, it would be preferable to report separate indicators:

- unique active participants;
- continuity of participation;
- diversity of participating people and Associations;
- inequality of influence;
- access to information;
- evidence that participation affected decisions.

A composite index may be explored later. The ability to write a formula does not justify combining unlike quantities.

A3.7 – A Small Example

Suppose the estimated unique-participant counts during four quarters are:

$$U_1 = 12,000, U_2 = 8,000, U_3 = 20,000, U_4 = 10,000.$$

The arithmetic mean is:

$$\bar{U} = \frac{12,000 + 8,000 + 20,000 + 10,000}{4} = 12,500.$$

The harmonic mean is:

$$\text{ENAP}_H = \frac{4}{1/12,000 + 1/8,000 + 1/20,000 + 1/10,000} \approx 11,163.$$

The harmonic mean is lower because low-participation periods receive greater importance.

Neither number is “the true amount of democracy.” The arithmetic mean describes average scale; the harmonic mean is more sensitive to low participation.

A3.8 – Repeated Participants and New Participants

The preceding formulas deduplicate people across Associations within each period. They do not distinguish repeated participation from the entry of different people over time.

An institution with 10,000 participants every quarter might involve the same people throughout the year or a much broader rotating population.

Additional quantities could include:

- U_{year} : the number of distinct people participating at least once during the specified year;
- D_t : the number participating in both period $t - 1$ and period t ;
- R_t : the proportion of the preceding period's participants retained in period t ;
- B_t : the number participating in period t who did not participate in period $t - 1$.

For $t \geq 2$, retention is:

$$R_t = \frac{D_t}{U_{t-1}}, U_{t-1} > 0.$$

The number entering relative to the preceding period is:

$$B_t = U_t - D_t.$$

Here, “entering” includes returning participants absent during the preceding period. First-ever participants would require a separate historical measure.

If the preceding period has zero participants, retention is undefined and should be reported as such.

High retention may indicate durable organisation, but also closure. High turnover may indicate broad reach, but also dissatisfaction or instability. None is automatically desirable.

A3.8.1 – Effective Sympathisers Between Elections

Opinion polls provide information different from membership and participation. Someone expressing support for a party is not necessarily active and may not vote for it. For a specified party or option, define:

- E_t : the number of eligible voters during period t ;
- s_t : the estimated proportion of eligible voters supporting it;
- p_t : the estimated probability that those supporters will vote;
- w_t : a non-negative weight assigned to the estimate.

Both s_t and p_t lie between zero and one. The weights may reflect recency or reliability, but their construction must be stated. At least one weight must be positive.

A rough estimate of likely supporting voters is:

$$S_t = E_t s_t p_t.$$

The symbol S_t denotes the period-specific estimate. It assumes that supporters who vote retain the stated preference. If the poll already estimates support among likely voters, a further turnout adjustment may be inappropriate.

A weighted arithmetic estimate across periods is:

$$S_w = \frac{\sum_{t=1}^T w_t S_t}{\sum_{t=1}^T w_t}.$$

Here, S_w is the weighted average of likely supporting-voter estimates.

A stability-sensitive alternative is:

$$\text{ENS}_H = \frac{\sum_{t=1}^T w_t}{\sum_{t=1}^T \frac{w_t}{S_t}}.$$

ENS means Effective Number of Sympathisers, and H identifies the harmonic-mean version.

Periods with zero weight are omitted. Positive-weight periods require positive support estimates. If one such period has zero support, this index is assigned zero.

The harmonic version penalises periods of low support. It should not be presented as the statistically preferred estimator of average electoral support. Poll aggregation also requires attention to sampling error, polling-house effects, time trends, turnout assumptions, and dependence among polls.

The two quantities answer different questions. The weighted arithmetic mean describes average estimated support; the weighted harmonic mean constructs an index sensitive to low-support periods.

Neither should be confused with party membership, active participation, or actual votes cast.

A3.8.2 – What ENAP Can Contribute

ENAP does not replace qualitative political judgement. It makes narrower comparisons possible:

- whether participation is increasing or decreasing;
- whether it is continuous or concentrated around elections and crises;
- whether reported membership greatly exceeds active participation;
- whether many membership records represent relatively few unique people;
- whether the same small circle participates repeatedly.

Alongside measures of internal democracy, influence, independence, and social diversity, **ENAP** may form part of an instrument panel for Associative democracy.

Its principal purpose is to make one question harder to avoid: when an institution claims to be participatory, how many distinct people actually participate, and how regularly?

APPENDIX A4 – Measuring the Effective Number of Economic Actors

A Statistical Reinterpretation of the Later Invisible-Hand Model

This appendix does not claim that Adam Smith proposed a statistical model of the invisible hand. As discussed in Section 15, the broader conception of markets as self-regulating systems developed through later interpretations.

My question is different: if markets coordinate dispersed economic decisions, how dispersed are those decisions?

The Effective Number of Economic Actors (**ENEA**) is a family of possible measurements. Market share, ownership, strategic independence, infrastructure, finance, and entry conditions are different dimensions. The formulas form an exploratory instrument panel, not a complete model of economic health.

Subscripts identify sectors, firms, or control groups. Superscripts written as words are labels, not powers. Numerical superscripts denote powers. Bold symbols represent vectors or matrices, and superscript ^{T} means transpose: a column vector is turned into a row vector, or vice versa.

A4.1 – Basic Setup

Assume that the economy under examination contains M sectors, indexed by:

$$j = 1, \dots, M.$$

Sector j contains n_j legal firms, indexed by:

$$k = 1, \dots, n_j.$$

Let y_{jk} be the selected measure of firm k 's activity in sector j . It might be sales, output, employment, assets, capacity, or another relevant quantity.

For share calculations, use non-negative quantities with a positive sector total. Define firm k 's sectoral share as:

$$s_{jk} = \frac{y_{jk}}{\sum_{l=1}^{n_j} y_{jl}}.$$

The index l runs over the same firms as k , but is used separately for summation. The shares satisfy:

$$\sum_{k=1}^{n_j} s_{jk} = 1.$$

The choice of activity measure matters. Employment, sales, assets, and capacity can produce different pictures of concentration. Every reported measure must identify its basis and observation period.

For economy-wide calculations, let q_j be the weight assigned to sector j , with:

$$q_j \geq 0, \sum_{j=1}^M q_j = 1.$$

Weights might reflect output, employment, assets, essential consumption, or another explicitly chosen basis. Different weights answer different questions.

A4.2 – Size-Adjusted Numerical Diversity

Define sector j 's share-concentration index as:

$$H_j = \sum_{k=1}^{n_j} s_{jk}^2.$$

Its inverse is:

$$N_j^{\text{share}} = \frac{1}{H_j}.$$

The superscript “share” identifies the share-adjusted number of firms: the number of equally sized firms producing the same concentration.

With ten equally sized firms:

$$H_j = 10(0.1)^2 = 0.1, N_j^{\text{share}} = 10.$$

If one firm holds 70 per cent and others divide the remainder, the share-adjusted number is much smaller than the raw count.

This improves upon counting company names, but does not establish independence. Equal-sized firms may share ownership, coordinate, or depend upon common infrastructure.

On the conventional HHI scale:

$$\text{HHI}_j = 10,000H_j,$$

where HHI means Herfindahl–Hirschman Index. Therefore:

$$N_j^{\text{share}} = \frac{10,000}{\text{HHI}_j}.$$

These conversions are mathematical. Judgements about healthy competition or democratic plurality are separate.

A4.3 – Ownership-Adjusted Diversity

Let ultimate ownership or control groups be indexed by $g = 1, \dots, G$, where G is the number of groups identified.

For this simple calculation, each firm is assigned to one control group, and all firms are covered. Independent firms can each constitute their own group. Overlapping or jointly controlled ownership requires additional treatment rather than repeated counting.

Define group g 's combined sectoral share as:

$$S_{jg} = \sum_{k \in g} s_{jk}.$$

The notation $k \in g$ means that the sum includes firms assigned to control group g .

Ownership-adjusted concentration and its inverse are:

$$H_j^{\text{owner}} = \sum_{g=1}^G S_{jg}^2,$$

$$N_j^{\text{owner}} = \frac{1}{H_j^{\text{owner}}}.$$

The label “owner” identifies adjustment for ownership or control groups.

Ten legal firms controlled by two groups may represent considerably less diversity than the firm count suggests.

Control is difficult to define. Majority ownership, voting agreements, minority holdings, creditor influence, and control over essential technology can matter in different ways. The measure therefore depends upon the stated control criterion and the quality of the data.

A4.4 – Strategic and Statistical Dependence

Different owners do not guarantee independent behaviour. Firms may coordinate, imitate one another, share suppliers, or respond to common shocks.

Collect sectoral firm shares in the column vector:

$$s_j = \begin{pmatrix} s_{j1} \\ \vdots \\ s_{jn_j} \end{pmatrix}.$$

Let R_j be the correlation matrix for a specified firm-level behaviour or outcome during a stated period. Its entry $\rho_{j,kl}$ is the correlation between firms k and l . Diagonal entries equal one.

An exploratory dependence-adjusted quantity is:

$$N_j^{\text{dep}} = \frac{1}{s_j^T R_j s_j}.$$

The label “dep” denotes dependence adjustment. The denominator must be positive.

When off-diagonal correlations are zero, the matrix is the identity matrix and the formula reduces to N_j^{share} . With equal shares, this equals the raw firm count. Zero correlation, however, does not necessarily establish statistical or strategic independence.

If all relevant outcomes are perfectly positively correlated, the quantity equals one.

For an interpretation as a reduction from share-adjusted diversity, this version should be used where off-diagonal correlations are non-negative. Negative correlations can produce values above the raw count, or a zero denominator. In that case, the formula describes statistical offsetting rather than a literal effective count of independent firms.

Its meaning depends upon the outcome measured, the period, and the stability of correlations. Common price movements may reflect coordination, shared costs, or common demand. Statistical correlation alone does not identify the cause.

The purpose is to examine dependence separately from market-share concentration—not to equate correlation with common control.

A4.5 – Variance Contributions

Suppose a sectoral aggregate is:

$$Y_j = \sum_{k=1}^{n_j} a_{jk} X_{jk},$$

where:

- Y_j is the aggregate random quantity;
- X_{jk} is a firm-level random variable;
- a_{jk} is its fixed aggregation weight.

These weights are not necessarily the market shares s_{jk} .

Let a_j be the column vector of weights and Σ_j the covariance matrix of the variables. Its diagonal entries are:

$$\sigma_{jk}^2 = \text{Var}(X_{jk}),$$

and its off-diagonal entries are covariances between firms. The notation Var means variance.

Then:

$$\text{Var}(Y_j) = a_j^T \Sigma_j a_j.$$

If covariance terms are zero, this becomes:

$$\text{Var}(Y_j) = \sum_{k=1}^{n_j} a_{jk}^2 \sigma_{jk}^2.$$

Provided this sum is positive, define:

$$c_{jk} = \frac{a_{jk}^2 \sigma_{jk}^2}{\sum_{l=1}^{n_j} a_{jl}^2 \sigma_{jl}^2}.$$

Here, c_{jk} is firm k 's share of the summed individual variance terms, excluding covariance. The shares sum to one.

Their concentration-adjusted number is:

$$N_j^{\text{variance}} = \frac{1}{\sum_{k=1}^{n_j} c_{jk}^2}.$$

The label "variance" identifies this particular basis of measurement.

Where firms are dependent, these terms do not allocate the whole aggregate variance. Covariances may be substantial and require separate examination. The formula isolates one dimension; it does not provide a complete dependence-adjusted measure.

A4.6 – The Central Limit Theorem

The Central Limit Theorem concerns conditions under which suitably centred and scaled sums of random variables approach a normal distribution.⁴

It does not establish that many firms must generate normally distributed prices, output, or profits. Nor does approximate normality establish market stability.

Application requires specification of the variables, aggregation rule, expected values, variances, dependence, and relevant limiting conditions.

Market prices are not ordinarily simple averages of independent firm-level variables. Firms respond to one another and to common conditions.

The theorem therefore supplies questions rather than proof: are contributions numerous, balanced, and sufficiently independent? Does one actor dominate? Are common shocks more important than individual variation?

These questions remain useful even where normality is neither expected nor observed.

A4.7 – Lindeberg's Condition

Lindeberg's condition supplies a sufficient condition for appropriately normalised sums of independent random variables to approach a normal distribution, including situations with unequal variances.⁵

⁴ Billingsley (1995).

⁵ Lindeberg (1922); Billingsley (1995).

Informally, the share of total variance associated with exceptionally large deviations must become negligible as the number of terms increases.

It does not specify a universal 20 per cent threshold for an actor's variance contribution. Failure of the condition does not, by itself, determine the resulting distribution—or even establish that normal convergence is impossible.

The 20 per cent value discussed in Section 15 may be retained as a proposed practical warning level, but not as a purported Lindeberg criterion. The quantity to which it applies must also be identified: a market share, an ownership share, and a variance-contribution share are different things.

Any such policy threshold is provisional, requiring empirical and sector-specific justification.

A4.8 – Distributional Diagnostics

Skewness, heavy tails, multiple modes, abrupt price changes, or increasing variance can supply useful information. They are not direct measures of competition or independence.

A dispersed market can produce skewed outcomes because demand is skewed. A concentrated market can maintain stable prices. Apparent stability may coexist with concentrated power, while variability may coexist with competition.

Distributional diagnostics should therefore be interpreted alongside ownership, market shares, dependence, entry and exit, finance, infrastructure, and institutional knowledge. Statistical shape may identify something worth investigating. It does not identify its cause on its own.

A4.9 – Economy-Wide Concentration

Sectoral and economy-wide concentration are different questions.

If actors are confined to separate sectors, a combined measure is:

$$N_{\text{economy}}^{\text{sector}} = \frac{1}{\sum_{j=1}^M \frac{q_j^2}{N_j^{\text{share}}}}.$$

The label “sector” identifies a calculation combining sectoral share-adjusted numbers. This is the inverse concentration of the combined actor shares $q_j s_{jk}$, under the assumption that no actor is counted as independent in several sectors. It is not an arithmetic average of sectoral effective numbers.

Cross-sector ownership often violates that assumption.

Using control groups $g = 1, \dots, G$, let w_{gj} be group g 's share of sector j . These shares must exhaust each sector without double counting:

$$\sum_{g=1}^G w_{gj} = 1.$$

Define group g 's economy-wide weighted share as:

$$G_g = \sum_{j=1}^M q_j w_{gj}.$$

Here, G is the number of control groups, whereas G_g is one group's weighted share. The shares satisfy:

$$\sum_{g=1}^G G_g = 1.$$

Economy-wide ownership concentration and its inverse are:

$$H_{\text{economy}}^{\text{owner}} = \sum_{g=1}^G G_g^2,$$

$$N_{\text{economy}}^{\text{owner}} = \frac{1}{H_{\text{economy}}^{\text{owner}}}.$$

A conglomerate spanning several sectors is counted as one control group.

Results depend upon sector definitions, control classifications, and weights. Employment, assets, output, and essential services may justify different versions. These differences should be visible rather than concealed in one apparently definitive number.

A4.10 – Entry, Exit, and Infrastructure

A high share-adjusted number can coexist with barriers to new entry.

The instrument panel should therefore include legal restrictions, capital requirements, credit, premises, intellectual property, network effects, data, platforms, essential suppliers, and access to consumers.

A platform may control the route to customers without holding a large direct share of sales. Revenue-based measures alone can understate its power.

Likewise, declining entry and continuing exits can warn of increasing concentration before a conventional numerical threshold is crossed.

A4.11 – Thresholds

No universal **ENEA** threshold separates healthy from unhealthy sectors.

Former American HHI classifications⁶ can be translated into inverse effective numbers:

$$\begin{aligned} \text{HHI} = 1,500 &\Leftrightarrow N^{\text{share}} \approx 6.67, \\ \text{HHI} = 2,500 &\Leftrightarrow N^{\text{share}} = 4. \end{aligned}$$

Here, $\Leftrightarrow$ indicates equivalent numerical conversions, and N^{share} is the share-adjusted number for the illustrative sector, with index j omitted.

These are not natural boundaries. They reflect guidelines from a particular period.

A democratic concern with dispersed power may justify different standards. Values such as 10, 30, or 100 should nevertheless be presented as proposed warning levels, not universal requirements.

Natural-monopoly conditions, infrastructure, systemic risk, innovation, and entry barriers differ among sectors. Thresholds must remain provisional, sector-specific, evidence-sensitive, and supplementary to institutional investigation.

A4.12 – Illustrative Comparisons

Bread production and automobile manufacturing illustrate the questions.

Many bakeries do not necessarily imply dispersed power throughout the production chain. They may depend upon a few flour suppliers, landlords, banks, platforms, or energy providers.

Automobile manufacturing has fewer large producers and substantial entry barriers. Yet brands may share owners, technologies, suppliers, finance, or state support.

Neither example establishes that one sector is healthy and the other unhealthy. They show why counting visible producers is insufficient.

A4.13 – Monitoring

A monitoring system could report raw firm counts, share-adjusted and ownership-adjusted numbers, estimated dependence, entry and exit, infrastructure control, financial relationships, relevant distributional diagnostics, and employee or consumer participation.

Economy-wide reporting could identify groups controlling activity across several sectors. The objective is not to eliminate every inequality of size. Large institutions can be useful, efficient, or necessary. The objective is to detect combinations of size, ownership, dependence, and control that place actors beyond effective competition and democratic accountability.

⁶ U.S. Department of Justice & Federal Trade Commission (2010).

A4.14 – Public and Collective Ownership

Public, municipal, cooperative, or communal ownership can counterbalance private concentration. It can also concentrate power in another institution.

Ownership form is therefore insufficient. A public enterprise may be authoritarian; a cooperative may be dominated by its leadership; and a private firm may allow meaningful employee participation.

Each must be examined through management, independence, accountability, and decision-making power.

The “Jupiter effect” remains a metaphor. Jupiter can stabilise some relationships while disturbing others. Public intervention likewise can protect essential services or prevent collapse, but can also suppress plurality.

Public actors belong within the economic system being measured. They are not automatic guardians standing outside it.

A4.15 – Relationship to ADM, ENDM, and ENAP

The concepts address related but different questions:

- **ADM** is the proposed Associative Democratic Model.
- **ENEA** concerns economic plurality, concentration, and independence.
- **ENDM** concerns the distribution of effective decision-making influence.
- **ENAP** concerns the scale and continuity of active participation.

Many firms can coexist with few effective decision-makers. Many participants can have little influence. Numerous Associations can share the same leadership.

These concepts should not be collapsed into one formula. Their usefulness lies partly in revealing different forms of concentration.

A4.16 – Summary

This proposal does not claim that many firms automatically produce stability, that normal distributions demonstrate competition, or that non-normal distributions demonstrate monopoly.

Its propositions are more limited: formal counts can exaggerate plurality; shares and ownership require separate measurement; dependence can further alter effective diversity; cross-sector control matters; and numerical indices require evidence about institutions, infrastructure, finance, and entry.

The inspiration comes partly from quantitative and evolutionary genetics, where effective numbers distinguish observed counts from the distribution and consequences of contributions. The transfer to economics cannot be mechanical.

No formula gives the whole picture. Several incomplete measurements, considered together, may help us judge whether an economy is moving towards greater plurality and democratic accountability or towards concentration and dependence.

That is the purpose of **ENEA**.

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