



Common statistical tools

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And, please remember that ...

Scientists are by no means bound to follow philosophers' advice on [conducting research], and they don't have such habits.

Jorjani (1995; *cf* Molander (1983))

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A word of caution

Don't believe whatever you read in Wikipedia!

I have my own opinion, and I don't give a damn to what Wikipedia or any other individual source says.

Form your own opinion

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Induction

Webster

An instance of reasoning from a part to a whole, or
A conclusion arrived at by reasoning from particulars to generals, or
Reasoning from the individual to the universal

Wiktionary

(logic) the derivation of general principles from specific instances
(mathematics) a general proof of a theorem by first proving it for a specific integer (for example) and showing that, if it is true for one integer then it must be true for the next

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Deduction

Webster

To deduce by logical process:

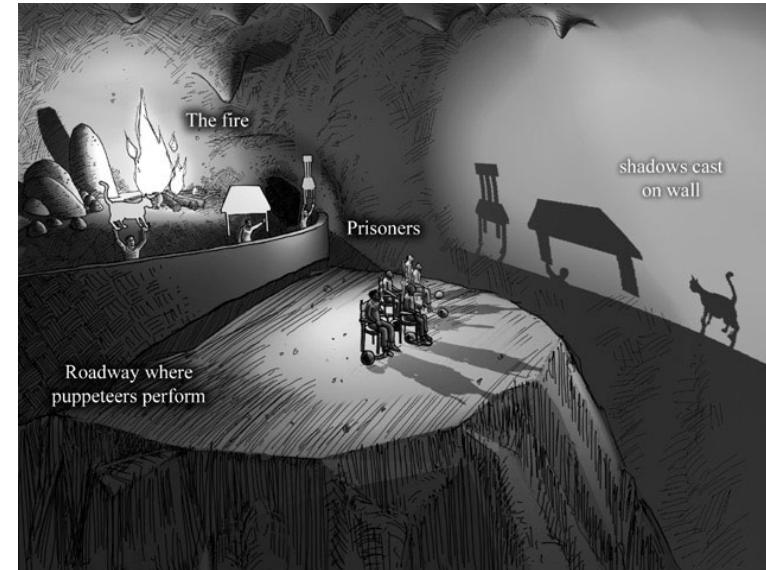
To draw (a conclusion) necessary from given premises;

To infer (something) about a particular case from a general principle that holds for all such cases.

Wiktionary

A process of reasoning that moves from the general to the specific, in which a conclusion follows necessarily from the premises presented, so that the conclusion cannot be false if the premises are true.

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Induction-deduction

Every phenomenon has a true type (archetype=idea).

The archetype can manifest itself in many forms.

Every phenomenon has an essence.

Essence will be known by examining many observations.



No more deduction, no more induction

Francis Bacon 1561-1626

We have suffered enough from deduction!

David Hume 1739-1740

Good science cannot be based on induction!

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What the hell are we going to do now?

We have suffered enough from deduction!

Good science cannot be based on induction!

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Summary

Two schools of statistics

Frequentist statistics

Bayesian statistics



Frequentist statistics

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Is this an inductive mode of reasoning?

Pearson, 1903

... probably the most correct way of looking at **any fraternal correlation** table would be to suppose it a random sample of all pairs of brothers which would be obtained by giving a large, or even indefinitely large, fertility to each pair, for what we actually do is to take families of varying size and take **as many pairs of brothers as they provide.**

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Is this an inductive mode of reasoning?

Fisher, 1918

... each **pair of brothers** is a random sample of two from an infinite fraternity, that is to say from all the sons which a pair of parents might conceivably have produced, ...

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Modern (frequentist) statistics

Regression

Francis Galton 1886

Correlation

(Francis Galton 1886)
Carl Pearson 1903

Analysis of variance

Ronald Fisher 1918

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Darwin's theories of evolution

FACT 1: Potential exponential increase of populations.
SOURCE: Paley, Malthus, ...



FACT 2: Observed steady-state stability of populations.
SOURCE: universal observations



FACT 3: Limitation of resources.
SOURCE: Observation reinforced by Malthus.



INFERENCE 1: Struggle for existence among individuals.
SOURCE: Malthus.



FACT 4: Uniqueness of the individual.
SOURCE: Animal breeders, taxonomists.



FACT 5: Heritability of much of the individual variation.
SOURCE: Animal breeders.



INFERENCE 2: Differential Survival, i.e. natural selection.
SOURCE: Darwin.



INFERENCE 3: Through many generations: **evolution**.
SOURCE: Darwin



- 1 - Evolution as such
- 2 - Evolution by common descent
- 3 - The origin of diversity
- 4 - Gradualness
- 5 - Natural selection

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Before Fisher (1918)

Regression & Correlation

No genetic theory

Based on phenotypic observations

Obviously normally distributed

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Mendel and evolution

Discrete distribution based on one single gene with large effect: “Hopeful Monster” theory

Continuous distribution with small gradual changes

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Biometrician-Mendelian controversy

Ronald Aylmer Fisher
(1890-1962)

Geneticist & Statistician



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Fisher (1918)

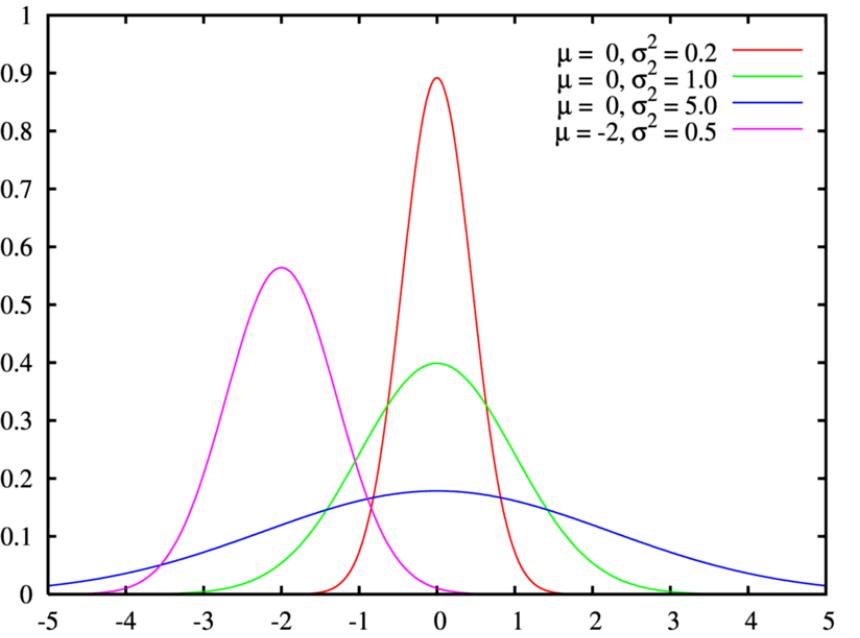
Analysis of variance

Based on Mendelian genetic theory

Postulating the aggregate behavior of many genes

Invoking normal distribution

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Inductive statistical Inference theory

Sir R. A. Fisher

(Repeated sampling)

Estimation of unknown parameters

Hypothesis testing

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Let's estimate some (sample) parameters

Mean

$$\bar{X} = \frac{\sum X}{n}$$

Variance

$$V_x = \frac{\sum (X - \bar{X})^2}{n-1}$$

Standard deviation

$$SD = \sqrt{V}$$

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Properties of variance

For independent X and Y

$$V_{(X+Y)} = V_X + V_Y$$

$$V_{(X-Y)} = V_X + V_Y$$

$$V_{(2X)} = 4V_X$$

$$V_{(kX)} = k^2 V_X$$

$$V_{(X+k)} = V_X$$

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Two equations for variance (actually SS)

$$\begin{aligned}
 \sum(X - \bar{X})^2 &= \sum(X^2 + 2X\bar{X} + \bar{X}^2) \\
 &= \sum X^2 - 2\bar{X}\sum X + n\bar{X}^2 \\
 &= \sum X^2 - 2\frac{\sum X}{n}\sum X + n\frac{\sum X}{n}\frac{\sum X}{n} \\
 &= \sum X^2 - \frac{(\sum X)^2}{n}
 \end{aligned}$$

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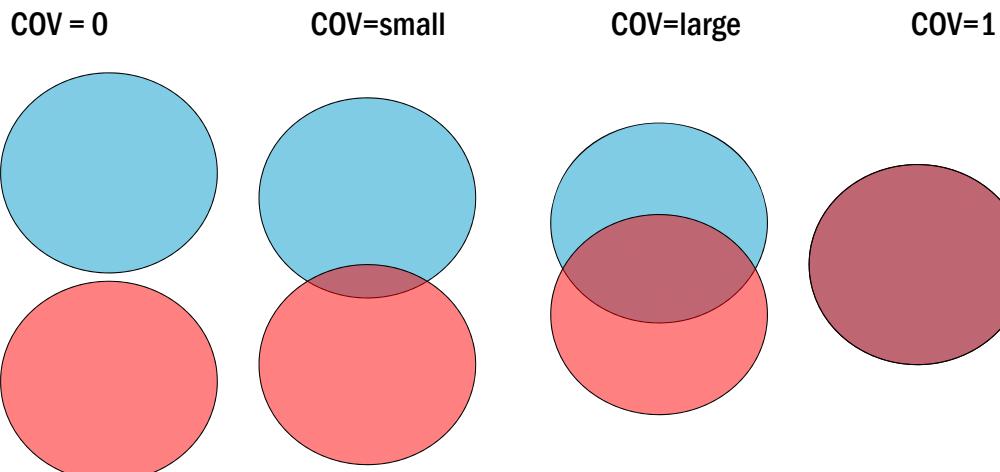
Regression and correlation

$$COV_{Y \square X} = \frac{\sum(X - \bar{X})(Y - \bar{Y})}{n - 1}$$

$$b_{Y \square X} = \frac{COV_{Y \square X}}{V_X}$$

$$r_{Y \square X} = \frac{COV_{Y \square X}}{\sqrt{V_X V_Y}}$$

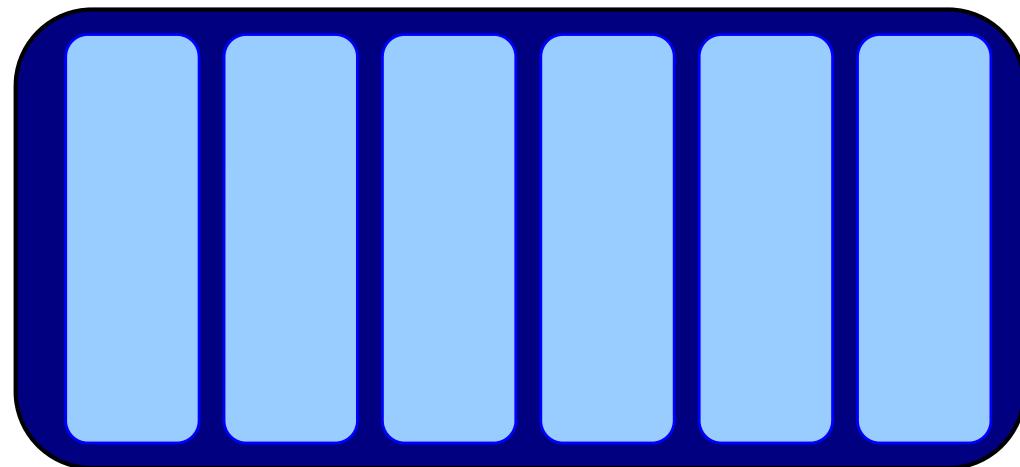
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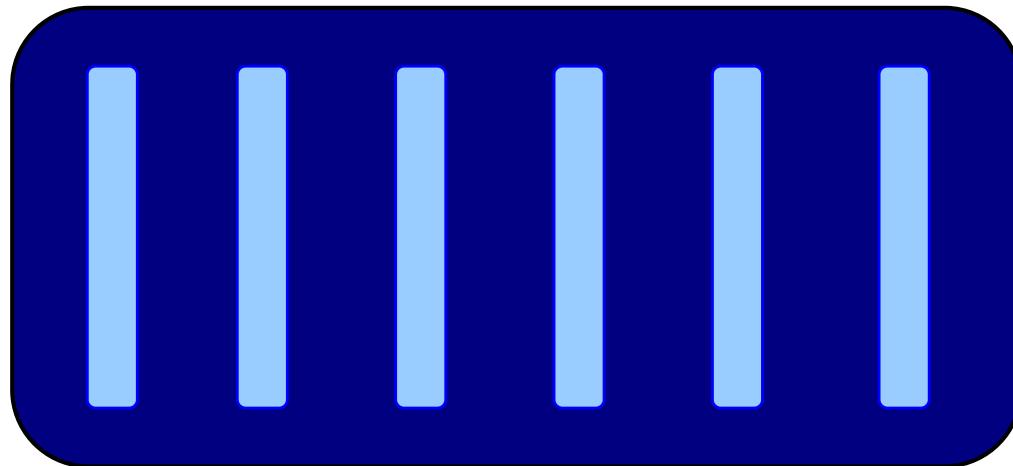
Within & Between group (co)variance



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Within & Between group (co)variance



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Standard what?

$$SE_{\bar{X}} = \sqrt{\frac{V_X}{n}}$$

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A simple hypothesis test

$$t = \frac{\bar{X} - \mu}{SE_{\bar{X}}}$$

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Modeling reality

Observations =

Nuisance parameters +

Parameters of interest +

Unexplained part

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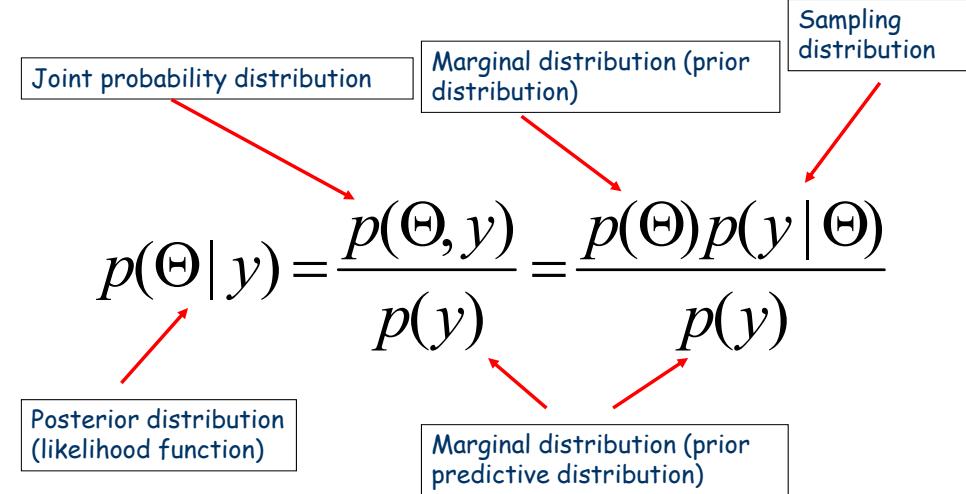


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Bayes' Rule, 1763 (1702-1761)



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