

# Common statistical tools



Hossein Jorjani  
hossein.jorjani@slu.se  
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## A word of caution

Don't believe whatever you read in Wikipedia!

I have my own opinion, and I don't give a damn to what Wikipedia or any other individual source says.

**Form your own opinion**

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## And, please remember that ...

Scientists are by no means bound to follow philosophers' advice on [conducting research], and they don't have such habits.

Jorjani (1995; *cf* Molander (1983))

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## Induction

### Webster

An instance of reasoning	}	from a part to a whole, or
A conclusion arrived at by reasoning		from particulars to generals, or
Reasoning		from the individual to the universal

### Wiktionary

(logic) the derivation of general principles from specific instances  
(mathematics) a general proof of a theorem by first proving it for a specific integer (for example) and showing that, if it is true for one integer then it must be true for the next

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## Deduction

### Webster

To deduce by logical process:

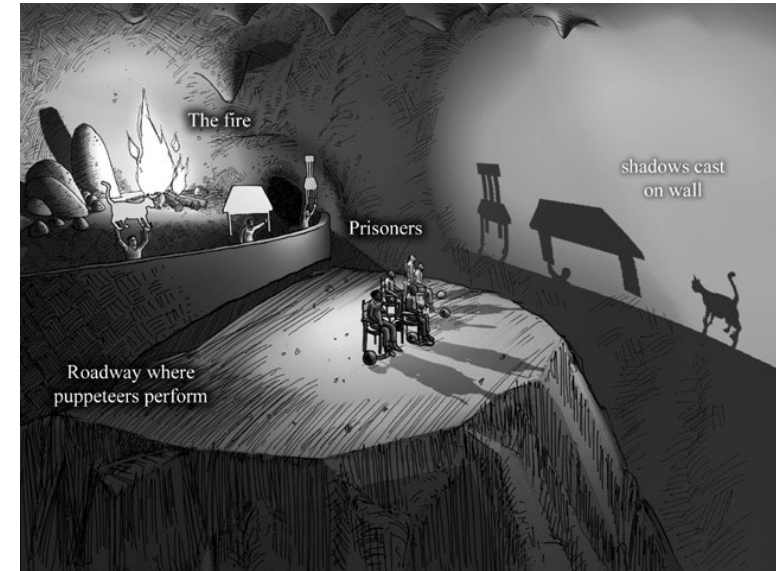
To draw (a conclusion) necessary from given premises;

To infer (something) about a particular case from a general principle that holds for all such cases.

### Wiktionary

A process of reasoning that moves from the general to the specific, in which a conclusion follows necessarily from the premises presented, so that the conclusion cannot be false if the premises are true.

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## Induction-deduction

Every phenomenon has a true type (archetype=idea).

The archetype can manifest itself in many forms.

Every phenomenon has an essence.

Essence will be known by examining many observations.

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## No more deduction, no more induction

Francis Bacon 1561-1626

We have suffered enough from deduction!

David Hume 1739-1740

Good science cannot be based on induction!

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**What** the hell **are we going to do now?**

**We have suffered enough from deduction!**

**Good science cannot be based on induction!**

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## **Summary**

**Two schools of statistics**

**Frequentist statistics**

**Bayesian statistics**

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**Frequentist statistics**

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## Is this an inductive mode of reasoning?

Pearson, 1903

... probably the most correct way of looking at **any fraternal correlation** table would be to suppose it a random sample of all pairs of brothers which would be obtained by giving a large, or even indefinitely large, fertility to each pair, for what we actually do is to take families of varying size and take **as many pairs of brothers as they provide**.

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## Is this an inductive mode of reasoning?

Fisher, 1918

... each **pair of brothers** is a random sample of two from an infinite fraternity, that is to say from all the sons which a pair of parents might conceivably have produced, ...

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## Modern (frequentist) statistics

Regression                      Francis Galton                      1886

Correlation                      (Francis Galton                      1886)  
   Carl Pearson                      1903

Analysis of variance                      Ronald Fisher                      1918

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## Darwin's theories of evolution

FACT 1: Potential exponential increase of populations.  
SOURCE: Paley, Malthus, ...



FACT 2: Observed steady-state stability of populations.  
SOURCE: universal observations



FACT 3: Limitation of resources.  
SOURCE: Observation reinforced by Malthus.



INFERENCE 1: Struggle for existence among individuals.  
SOURCE: Malthus.



FACT 4: Uniqueness of the individual.  
SOURCE: Animal breeders, taxonomists.



FACT 5: Heritability of much of the individual variation.  
SOURCE: Animal breeders.



INFERENCE 2: Differential Survival, i.e. natural selection.  
SOURCE: Darwin.



INFERENCE 3: Through many generations: **evolution**.  
SOURCE: Darwin



- 1 - Evolution as such
- 2 - Evolution by common descent
- 3 - The origin of diversity
- 4 - Gradualness
- 5 - Natural selection

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## Before Fisher (1918)

### Regression & Correlation

No genetic theory  
Based on phenotypic observations  
Obviously normally distributed

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## Mendel and evolution

Discrete distribution based on one single gene with large effect: “Hopeful Monster” theory

Continuous distribution with small gradual changes

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## Biometrician-Mendelian controversy

Ronald Aylmer Fisher  
(1890-1962)

Geneticist & Statistician



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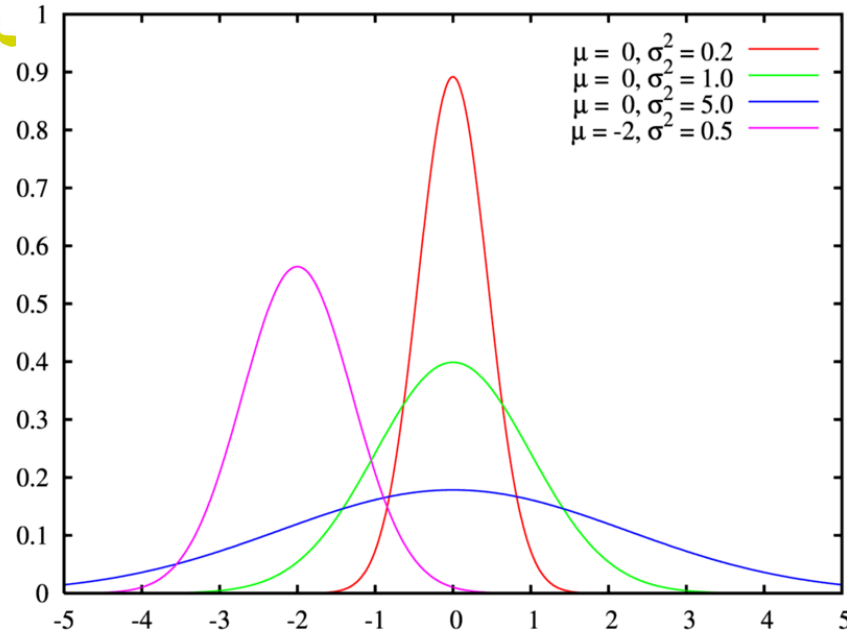


## Fisher (1918)

### Analysis of variance

Based on Mendelian genetic theory  
Postulating the aggregate behavior of many genes  
Invoking normal distribution

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## Inductive statistical Inference theory

Sir R. A. Fisher

(Repeated sampling)

Estimation of unknown parameters

Hypothesis testing

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## Let's estimate some (sample) parameters

Mean

$$\bar{X} = \frac{\sum X}{n}$$

Variance

$$V_X = \frac{\sum (X - \bar{X})^2}{n-1}$$

Standard deviation

$$SD = \sqrt{V}$$

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## Properties of variance

For independent X and Y

$$V_{(X+Y)} = V_X + V_Y$$

$$V_{(X-Y)} = V_X + V_Y$$

$$V_{(kX)} = k^2 V_X$$

$$V_{(kX)} = k^2 V_X$$

$$V_{(X+k)} = V_X$$

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## Two equations for variance (actually SS)

$$\begin{aligned}
 \sum (X - \bar{X})^2 &= \sum (X^2 + 2X\bar{X} + \bar{X}^2) \\
 &= \sum X^2 + 2\bar{X} \sum X + n\bar{X}^2 \\
 &= \sum X^2 + 2\frac{\sum X}{n} \sum X + n\frac{\sum X}{n} \frac{\sum X}{n} \\
 &= \sum X^2 - \frac{(\sum X)^2}{n}
 \end{aligned}$$

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## Regression and correlation

$$COV_{Y \square X} = \frac{\sum (X - \bar{X})(Y - \bar{Y})}{n - 1}$$

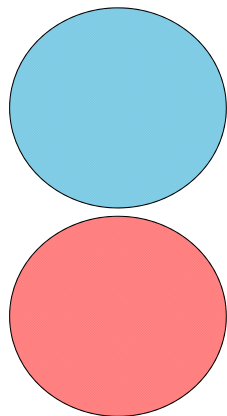
$$b_{Y \square X} = \frac{COV_{Y \square X}}{V_X}$$

$$r_{Y \square X} = \frac{COV_{Y \square X}}{\sqrt{V_X V_Y}}$$

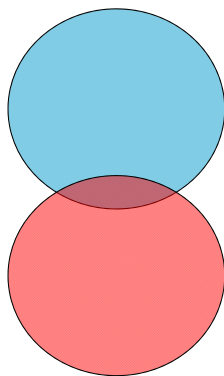
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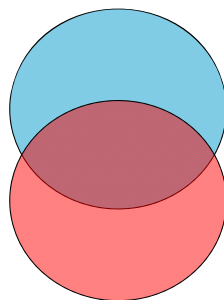
COV = 0



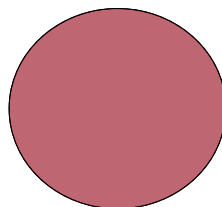
COV=small



COV=large



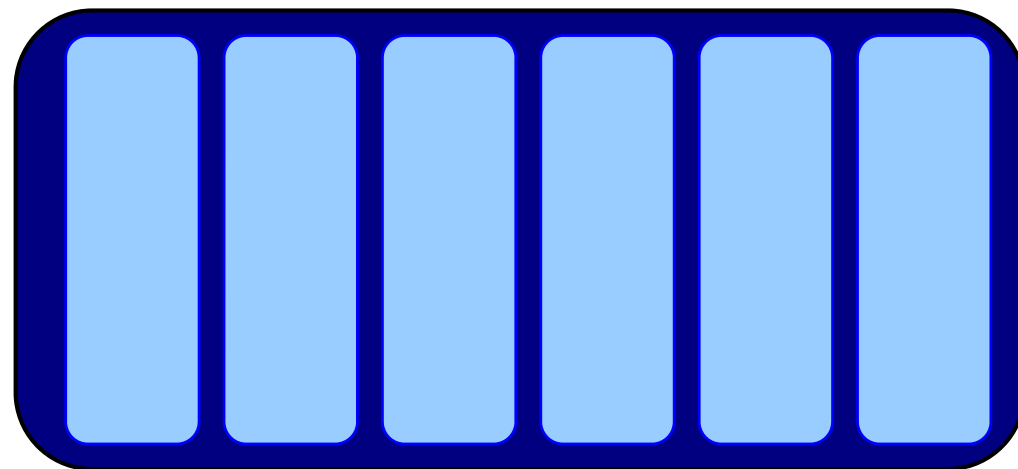
COV=1



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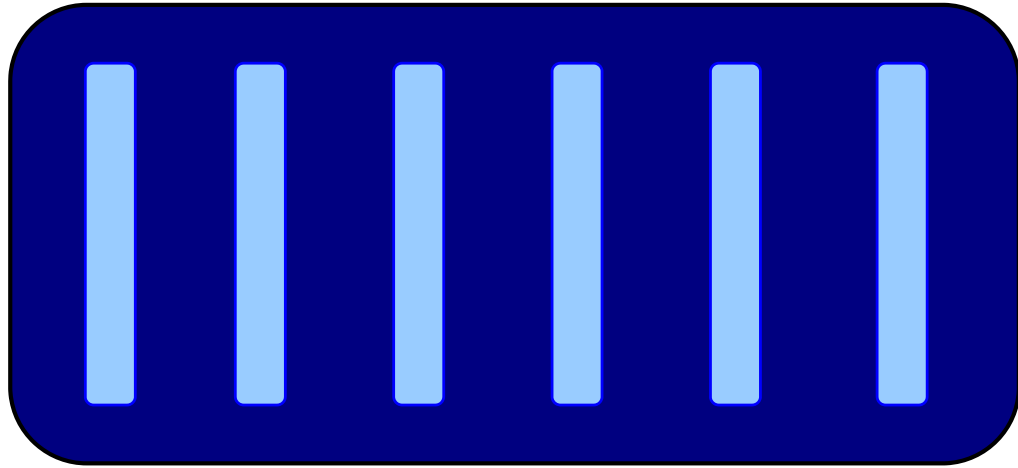
## Within & Between group (co)variance



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## Within & Between group (co)variance



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## Standard what?

$$SE_{\bar{X}} = \sqrt{\frac{V_X}{n}}$$

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## A simple hypothesis test

$$t = \frac{\bar{X} - \mu}{SE_{\bar{X}}}$$

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## Modeling reality

Observations =

Nuisance parameters +  
Parameters of interest +  
Unexplained part

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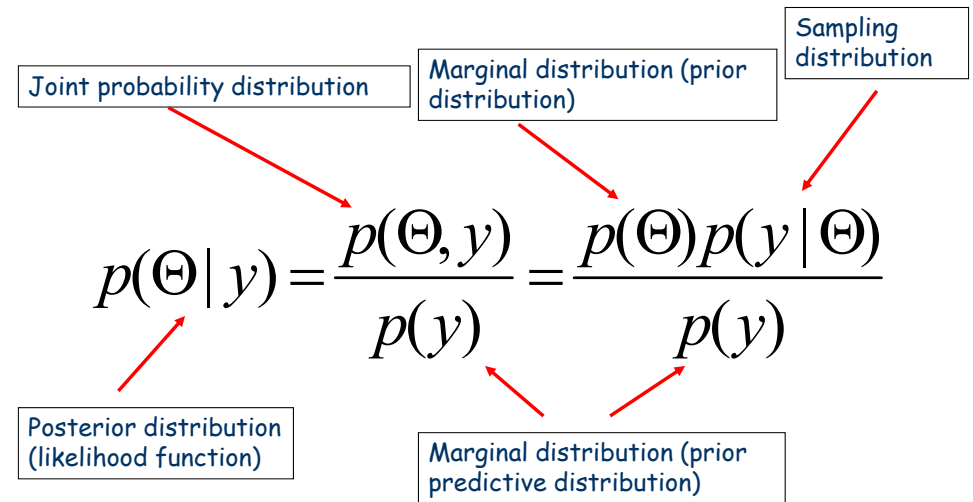


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## Bayes' Rule, 1763 (1702-1761)



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Hossein Jorjani

Department of Animal Breeding & Genetics  
Swedish University of Agricultural Sciences  
Box 7023  
S – 750 07 Uppsala, Sweden

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